A Mean-Variance Approach for Optimizing Physical Commodity Production Decisions

Tom Soutter
Trader, Fonterra Co-operative (New Zealand)

Isaac Manuel
Trader, Fonterra Co-operative (New Zealand)

Introduction

In general, processors aim to maximize their profit by operating up to the point where their marginal cost equals their marginal revenue. This is challenging to do when supply and demand are volatile, which is a dynamic often observed in commodity markets. Common optimization planning models can address these scenarios by forecasting sales prices and customer demand, then running iterative algorithms over the data to generate the production plan that maximizes profit at a point in time. However, these production plans can be highly reactive as product price forecasts and other inputs change. This paper examines an approach to optimizing physical production decisions ("make") that considers risk, specifically in situations where:

1. The processor/refiner converts one raw input into many output combinations. For example, milk gets turned into a set of products, such as a combination of cheese and whey protein;

2. The processor is constrained by the supply of the raw input. These constraints could arise from geographic supply limitations, regulation, or agreements with suppliers;

3. The processor has plant capacity limitations and cannot convert the raw input into only one product-mix (that is, some undesirable products may need to be made due to the location/availability of physical refining assets);

4. Processors may be forced to make certain products, regardless of their margins, due to contracts with customers; or

5. Processors make products that either cannot be hedged or that do not have an appropriate liquid hedging instrument.

This report investigates an alternative approach to optimizing make using mean-variance analysis. Markowitz's mean-variance analysis is a well-known method for asset portfolio optimization that considers both risk and return (Markowitz, 1952). By applying this framework, an alternative approach to commonly used optimization planning models is taken. It is hypothesized that the advantages in this framework are a more consistent make over the long term and less volatile returns. The following sections discuss a method for adapting the Markowitz theorem to a constrained processor situation where the processor takes a raw input and has the option to refine it into many products, and finally an example is presented.

Method

Markowitz's mean-variance analysis provides the portfolio weightings, $\mathrm{w_i}$, for asset, i, that maximize the expected excess return per unit of standard deviation. These weightings are calculated from the returns, $\mathrm{R_i}$, and covariance, $\rho_{\mathrm{i,j}}$, of assets i and j. In this application i and j are streams and $\mathrm{R_i}$ is the expected average return of the stream, i. Absent any constraints, one unit of raw input collected by a commodity processor can be transformed into any stream of products. Consider a stream as the set of products made by a recursive procedure utilizing the raw input and the subsequent by-products of production. As an example, when skim milk is extracted from raw milk, a fat-concentrated liquid remains. From this, the fat can be extracted and butter produced, leaving butter milk as the remaining by-product. Therefore, an example of a stream is: $i = \{skim\ milk, butter, butter\ milk\}$.

The value of a stream can be calculated at any point in time. This value is the sum of the yield-weighted prices of all component products in the stream minus their yield-weighted marginal production costs. Return, $\mathrm{R_i}$, is the percentage change in value of the stream from period $t-1$ to period t, which is shown in the equation below:

$$\mathrm{R_i} = \sum_k \alpha_{i,k} \frac{(P_{k,t}-C_{k,t})-(P_{k,t-1}-C_{k,t-1})}{(P_{k,t-1}-C_{k,t-1})}$$

where $\mathrm{P_{k,t}}$ is the price of the commodity, $k \in i$ at time t, and $C_{k,t}$ is the variable production cost of k at time t. $\alpha_{i,k}$ is a yield coefficient, $k \in i$, such that if the processor takes one unit of the raw input they will make $\alpha_{i,k}$ units of the commodity k. Likewise, in this scenario $\rho_{\mathrm{i,j}}$ becomes the covariance between the returns of streams i and j. Next, the processor's constraints are discussed.

If the processor operates in a commodity where the refined products do not have a liquid financial market or if the processor does not have access to the financial markets then the solution is constrained to have weightings $\mathrm{w_i} \geq 0$ for each stream, i. This is important as the Markowitz optimal portfolio may suggest that the processor should short some streams.

Processors also have capacity and production constraints. Capacity restrictions, such as physical refining limitations, are likely to cause minimum and maximum output constraints for certain products. Production constraints may force the processor to make some products regardless of whether this is optimal in the mean-variance sense. The minimum make, m_k, and maximum make, M_k, are dependent on the unique situation of a given processor. That is, the make v_k , $m_k \leq v_k \leq M_k$. For example, supply agreements may force the make of the downstream product k to be greater or equal to some minimum make, m_k; and plant capacity constraints may force the make of k to be less than or equal to some maximum make, M_k.

Once the relevant constraints have been established the optimal portfolio can be computed using an optimizer such as the Excel Solver. The optimizer would assign a portion of the total raw input, U, to the streams. The make of product k in a stream i is,

$$v_{k,i} = U\beta_i\, \alpha_{i,k}.$$

where β_i is the proportion of raw input assigned to stream i such that $1 = \sum_i \beta_i$ (that is 100% of the raw input is assigned to a stream.) It follows that the total make across all streams is, $v_k = \sum_i v_{i,k}$, which must be within the make constraints.

The optimizer must allocate the constrained raw input across the streams such that it maximizes the Sharpe ratio. It is important to note that β_i is not analogous to the asset weightings in mean-variance analysis. In this framework, the weights, w_i, represent the proportion of the total revenue that is expected to come from each stream,

$$w_i = {\sum_k P_k v_{i,k}} \Big/ {\sum_k P_k, v_k}$$

$$\mu = \boldsymbol{w}^T \boldsymbol{R},$$

$$\sigma^2 = \boldsymbol{w}^T \boldsymbol{\rho}\, \boldsymbol{w},$$

$$\text{Sharpe ratio} = \frac{\mu - r}{\sigma},$$

where $\boldsymbol{w}$ is the vector of weights, $\boldsymbol{R}$ is the vector of returns, $\boldsymbol{\rho}$ is the stream covariance matrix, and r is the risk-free rate.

An Example: Applying This Method to a Lifelike Dairy Company

Figure 1
Sample Yield Table Applied in Dairy Processor Example

	kg of product per kg of base product processed				
Base Product	Butter	Butter Milk Powder	Whey Protein Concentrate	AMF/Ghee	Lactose
Whole Milk Powder	0.03				-0.04
Skim Milk Powder	0.44	0.01			-0.08
Cheese-Dry Salted		0.09	0.02	0.01	0.27
Casein	1.58	0.03	0.03		0.25
MPC 70	0.96	0.01			0.98

The above yield table was derived using data from Sneddon *et al.* (2015) and Bylund (2003). Using an extended version of this table, a historical time series of stream values was calculated with the time period of the underlying proprietary price series running from July 2005 through July 2017. The period-to-period percentage change in these stream values was taken as the return time series to be used in Markowitz's mean-variance framework, $\mathrm{R_i}$, and the annual variance-covariance matrix between streams was calculated.

In the optimal case no further constraints would be applied from this point onward. For a set of product prices chosen by the processor, the model would calculate the weight in each stream required to get the processor to the tangency portfolio (the orange point in the chart in Figure 2 on the next page) where the Sharpe Ratio is maximised. When the model was run with only a raw input constraint and no make constraints, a Sharpe ratio of 0.63967 was achieved. This is 0.10% below the theoretically optimal Sharpe ratio of 0.64034. It should also be noted that this solution involved shorting some products.

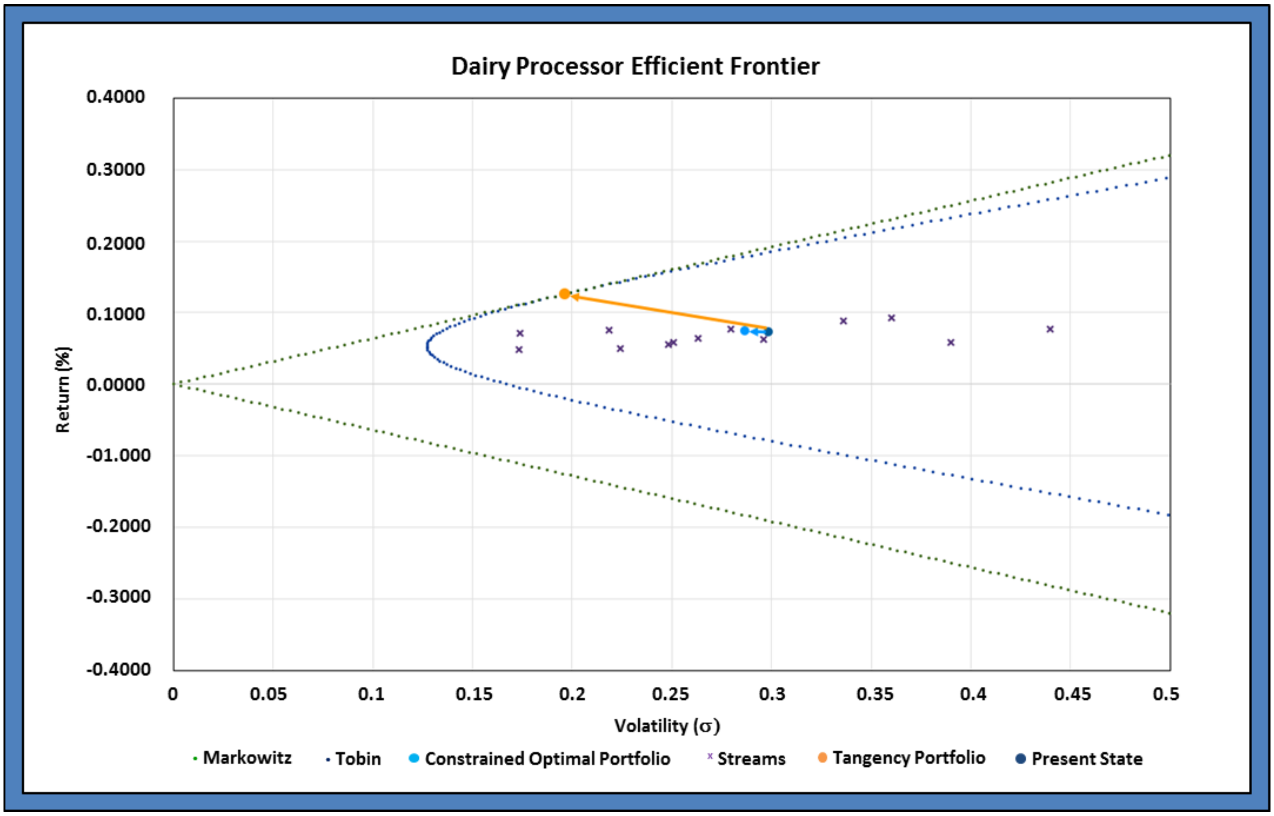
Setting Up the Minimum and Maximum "Forced Make" Production Constraints

The upper production constraints, M_k, for each product were estimated by, $\widehat{M}_k$, using the historical maximum production for each product in a given month scaled up by 10%. This upper bound is theoretically less than the maximum refining capacity of the dairy company. However, this estimation was used because milk refining is geographically constrained and therefore the maximum refining capacity is not a realistic upper bound.

The lower bound, $\widehat{m}_k$, assumed no forward sales contracts and estimated the minimum make under the assumption that milk was geographically constrained. It is assumed that all products had infinite demand, the prices of all alternative products were at their 95th percentile, and that milk collections were 15% below forecast. In other words, the forced make of an undesirable product in a year where milk solids collected were low was considered.

To calculate the constrained optimal portfolio (the blue point in the chart in Figure 2), Excel's GRG Nonlinear Solver was used. The Solver was set to find the make, v_k, of each product that led to the set of weights, $\mathrm{w_i}$, which maximized the Sharpe ratio of the portfolio with the constraint $\widehat{m}_k \leq v_k \leq \widehat{M}_k$ applied. This procedure was run several times and with varying initial criteria and mutation rates; each run resulted in the same optimal output.

Figure 2
Efficient Frontier Output as Applied to a Dairy Processor



Conclusion

This paper presents a risk aware framework for physical production planning for a commodity processor that steps away from traditional optimization approaches. This approach treats production decisions as analogous to a fund manager's asset selections where the processor's universe of assets is the streams of products that it can make. By applying mean-variance analysis it is expected that a processor will be more fairly rewarded for the risk implicit in their production plan.

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Author Biographies

TOM SOUTTER
Trader, Fonterra Co-operative (New Zealand)

Mr. Tom Soutter is a trader at Fonterra in Auckland, New Zealand. He focuses on market-making in NZX Dairy Commodity and Milk Price Derivatives, using these markets to help manage the risk of Fonterra's physical exposures. He has a B.Sc. in Mathematics and Economics and a B.Com. in Finance from the University of Otago. Mr. Soutter passed the CFA Level III in 2018.

ISAAC MANUEL
Trader, Fonterra Co-operative (New Zealand)

Mr. Isaac Manuel is a trader at Fonterra in Auckland, New Zealand. He manages Fonterra's Energy and Environmental portfolios. In 2013 he graduated first-in-class from the University of Canterbury in Computer Engineering and was awarded a Fulbright Science and Innovation Award. Following university he competed in triathlons until joining Fonterra in 2017.