Negative Oil Prices, Options, and the Bachelier Model

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The oil market has gone through a tumultuous period in early 2020. The price of the West Texas Intermediate Blend hit a peak of over $60 per barrel and then plunged for the first time in history to a negative price for both the front month future (CLK0) and spot price at Cushing on 4/20/2020. This paper focuses on the apparent stability of the market during this time period and the financial engineering challenges that options and futures traders addressed to ensure the markets remained orderly and operating. We provide evidence that the market functioned normally in the face of a negative futures price (CLK0) and the listing of negative strike options. We specifically focus on the difficulties in pricing and hedging of options under the traditional Black option model. Then, we explore two alternative model formulations and comment on their applicability.

Background

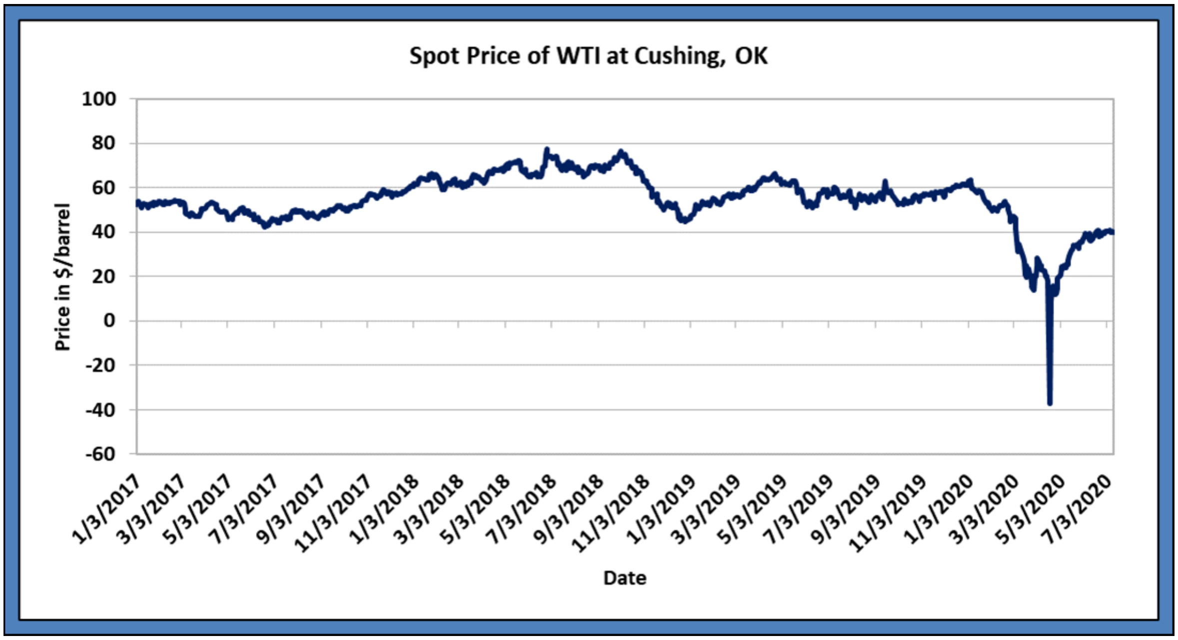
The "oil market" in the public's mind is a monolithic one, with one price quoted per barrel. The reality of the situation is that there are a multiplicity of oil standards and benchmarks. These benchmarks are driven by geography, oil composition, and market needs. The most widely followed benchmark is the so-called West Texas Intermediate (WTI) Blend. In recent years, there are other widely disseminated benchmarks, ranging from the Brent North Sea Crude to the Urals Blend. Each price per barrel reflects a unique supply and demand curve at a specific regional market and a specific point in time.

Additionally, there are different segments to this market. There is a "spot" market where oil is traded for immediate delivery. Most commentators will typically conflate the price of the spot market in Cushing with that of the WTI future, which trades on the NYMEX exchange. In Figure 1, we show the time series of the West Texas Intermediate Blend spot price at Cushing, Oklahoma. Prior to April 20, 2020, the price had a range of $10.25 to $145.31. By any measure, the price of oil is quite volatile. It has a standard deviation of 1.4 when viewed as daily price changes or 2.75% when expressed as a return (see Table 1).

Table 1
Summary Statistics of the WTI Crude Price

	Overall From Jan 2, 1986 to July 13, 2020			Prior To April 20, 2020			Month After April 22, 2020		
	Raw Price	Changes in Price	Changes in Log Price	Raw Price	Changes in Price	Changes in Log Price	Raw Price	Changes in Price	Changes in Log Price
Min	-36.98	-55.29	-40.64%	10.25	-14.76	-40.64%	12.17	-3.82	-27.30%
Max	145.31	45.89	42.58%	145.31	18.56	37.47%	34.30	4.73	42.58%
Average	32.81	-0.03	0.23%	44.14	0.00	0.00%	23.39	1.07	5.76%
Std Dev	29.06	1.40	2.73%	29.12	1.17	2.64%	7.12	1.88	12.70%
Std Dev Ann		22.24	43.33%		18.53	41.89%		29.89	201.61%

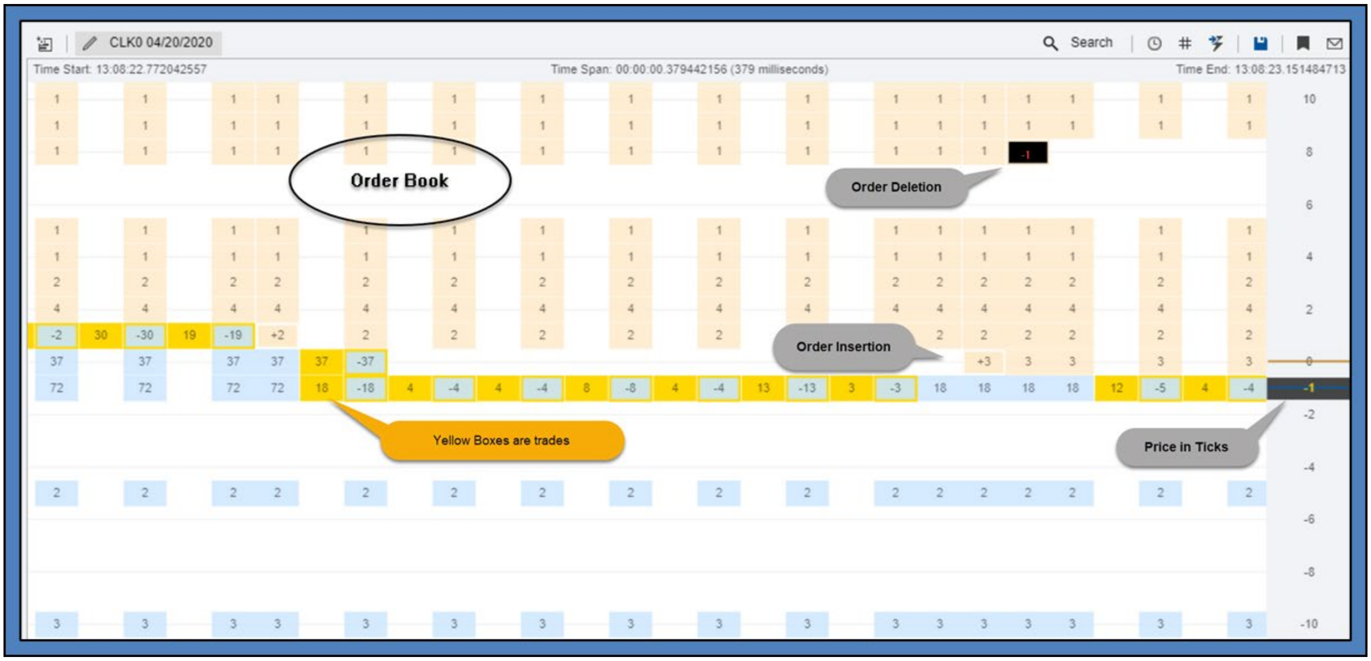
Figure 1
The Price of the West Texas Intermediate (WTI) Blend at the Cushing, OK Delivery Point



The price of oil is very volatile because of many factors, ranging from the fickleness of end-user demand to limited storage facilities, and producers unable to slow the delivery of oil due to complexity of shipping oil via pipelines and ships. However, prior to April 20, 2020, the price of a *major benchmark* for oil was never observed to be negative.[1] Previously, it had always been assumed that there was a lower bound to prices due to the fact that producers could stop producing when it became marginally uneconomical. Nonetheless, on April 20, 2020, the expiring May crude oil future (CLK0) at the New York Mercantile Exchange traded down to a negative price of -$37.63. The spot price at Cushing also fell to a negative price of -$36.58. Amongst the causes of the negative prices was limited available storage, as discussed in EIA (2020). It would be tempting to believe that such a colossal plunge in the market is indicative of a break down or flash crash.

Using data from Vertex Analytics in Figure 2 on the next page, we see that the microstructure shows an orderly market with books transitioning through the zero boundary as normally as they might transition through any other positive price level. The order book is always present and does not disappear. In fact, 37 futures contracts traded on the move through zero, with an additional 18 contracts trading immediately after the market prices broke into negative territory.

Figure 2
A Screenshot from Vertex Analytics Showing the Microstructure Around the Time the Oil Futures Contract Transitioned to Negative Prices

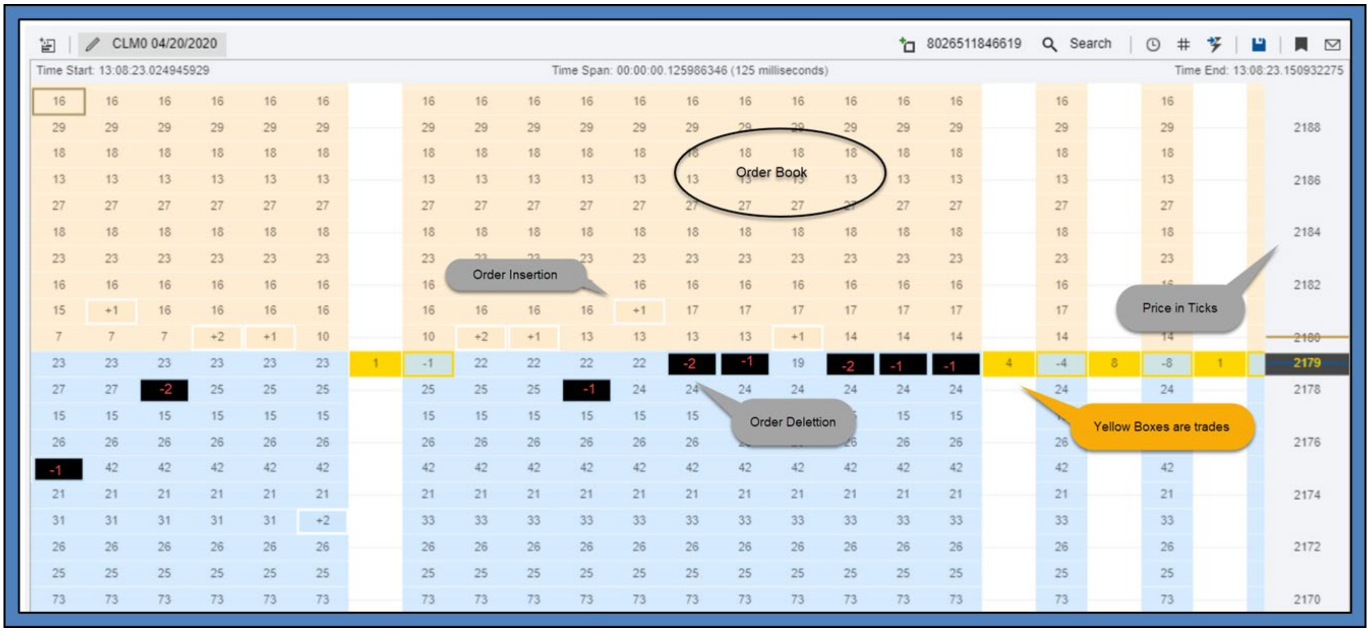


The screen is courtesy of Vertex Analytics.

Notes: Each column represents the order book at each update to the order book. The yellow blocks are the actual trades. The items in blue immediately behind the yellow block are the trade confirmation and the adjustment to the order book. Resting offers in the market are represented by a black number with a beige background. Resting bids are represented by black numbers with a blue background.

Figure 3 on the next page presents the June futures market state as May transitioned over the 0 price threshold. Note that the book is full. There is nothing to suggest a panic. This was a futures market that was not capitulating as an institution because of an unforeseen shock. Market participants' behavior could lead to the assumptions that at least the market makers were prepared for negative oil prices, even if market pundits were not.

Figure 3
The Market State in the June Crude Oil Future

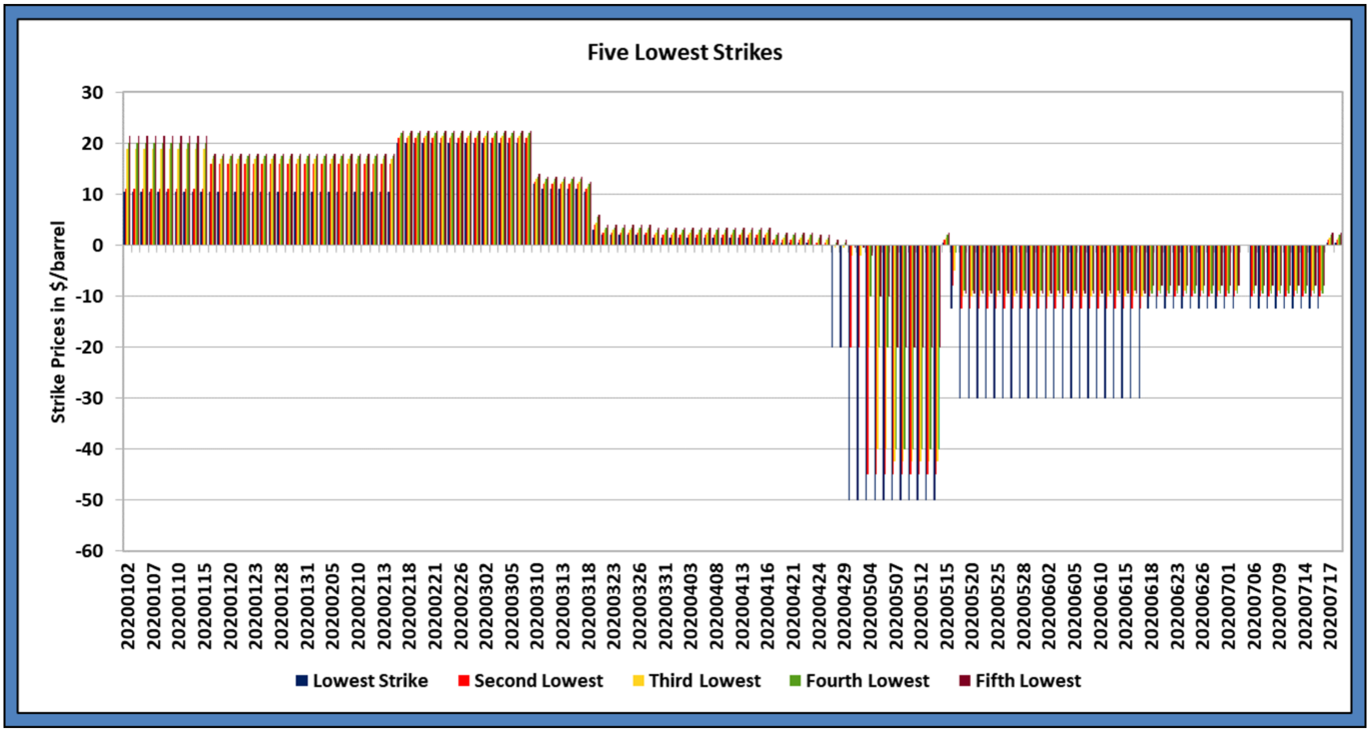


The screen is courtesy of Vertex Analytics.

Note: This is a snapshot of the June crude oil future as May transitioned into negative territory. Notice that the books remain full and there is no erosion in the size.

On the options side of the world, the best evidence that the market functioned was the behavior of the listed strikes during this move. To be clear, the options tied to the May contract (CLK0) had expired one week before the futures went negative. They never directly experienced that shock. However, if we look at the lowest strike prices with traded volume this year (Figure 4 on the next page), we see that they followed the evolution of prices smoothly. The first sub-10 dollar strike trades on March 19, with a negative 20 dollar strike trading on April 28th. The negative strikes trade until July 17th. Trading in negative strikes was real and persisted. Finally the futures options markets continued to function normally with negative strikes for months after the April 20th event. It is clear by the market microstructure behavior that market participants were prepared and had functional option pricing models to price negative strikes prior to April 20th.

Figure 4
The Five Lowest Traded Strikes for Front Month Options in WTI Crude (CL)



The Problem

The options market in crude oil is (primarily) a futures option market. A common way to price these options is through the use of the so-called Black model (Black, 1976) and its derivatives (Barone-Adesi and Whaley, 1987 or Bjerksund and Stensland, 1993). The Black model is a specialization of the Black-Scholes model (Black and Scholes, 1973), which recognizes that the futures price is approximately driftless and so we can write (to price a call with puts following from put/call parity):

$$C = e^{-rt}\{F\Phi(d_1) - K\Phi(d_2)\} \tag{1}$$

K	*strike price*
F	*futures price*
σ	*implied volatility*
r	*interest rate*
t	*time*
Φ	*Standard Normal Distribution*
d_1	$\dfrac{\ln(F/K) + (\sigma^2/2)T}{\sigma\sqrt{T}}$
d_2	$d_1 - \sigma\sqrt{T}$

Futures options in the U.S. are mostly American style expiry, so Black's model is an approximation. We choose the Black model because of its analytical tractability and ease of exposition. There were a couple of major shortcomings with the Black formula in the run up and aftermath of the negative prices. We detail those in the next few paragraphs.

The first obvious problem is the existence of a singularity when the asset price drops to or below 0. The logarithm is not defined there and as a result there would be no defined price. Thankfully, this occurred on only one day in the period and that was after the May options had expired.[2] The next concern occurs with very low (but not zero) prices - both for the futures price and the strike price. When either is very small, the sensitivity of the option's price to the change in implied volatility, the so-called Vega, diminishes dramatically. This can be seen in either formulation for the Vega.

$$vega = F e^{-rt} \phi(d_1) \sqrt{T} = K e^{-rt} \phi(d_2) \sqrt{T} \qquad (2)$$

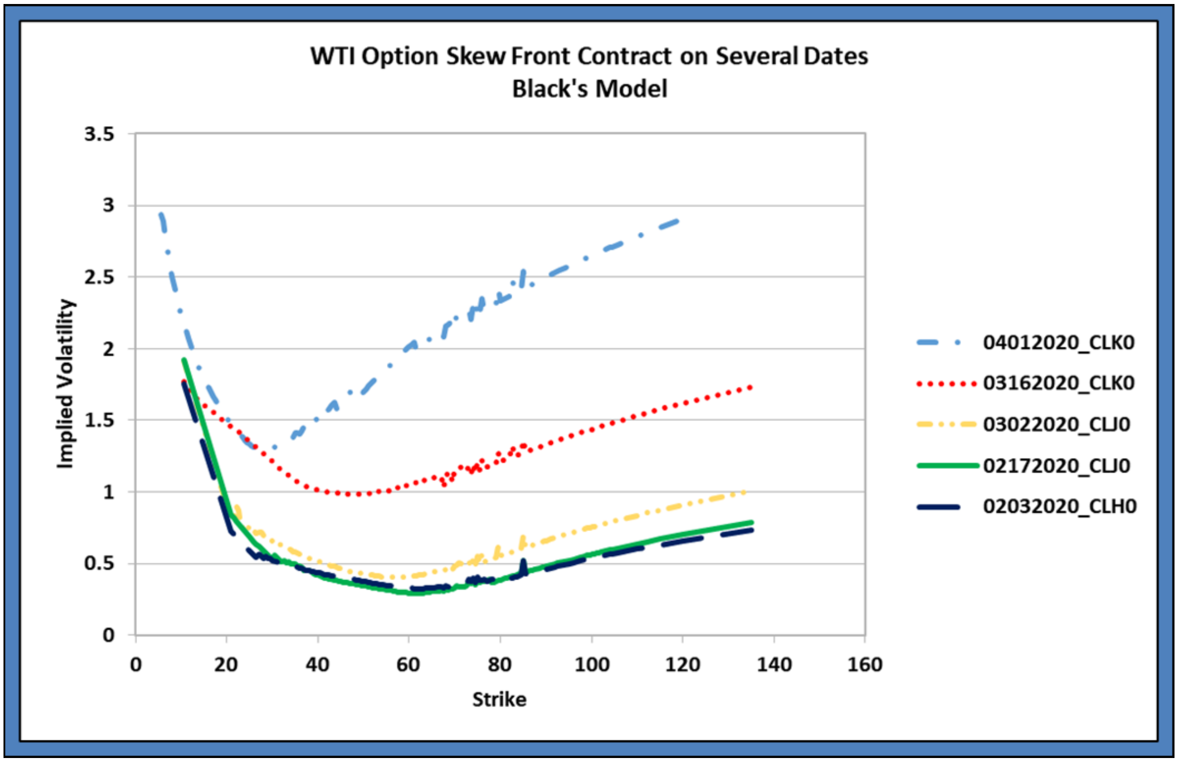
This is a key problem. As the prices decline to near zero, in order for the calls to retain any value, the implied volatility must increase dramatically. Put differently, as the price approaches the absorbing boundary, the moves (anticipated absolute dollar changes) would become smaller according to the model. The moves are limited by zero to the downside and some return on a very small base to the upside.

The fact is that dollar moves in oil did not attenuate as the front futures traded below ten dollars. In the month after the negative settlement, the standard deviation of the dollar move was higher (at 1.8) than it had been in the period prior to the debacle (at 1.07). The daily standard deviation of the return exploded to 12.7% from 2.64%. Clearly, the dollar risk would seem to be a more stable measure of risk.

Two things would affect the option prices. First, since the dollar changes did not attenuate, a log normal model would need to compensate by assuming a higher implied volatility. The dollar moves increased by a factor of 80%, and the return's standard deviation exploded by 600%. Second, at lower prices and strikes, the Black price is insensitive to implied volatility. To account for this insensitivity, option market makers would need to set the implied volatility curve for the Black option pricing model at extreme values, as shown in Figure 5 on the next page.[3] An option seller would have no choice but to increase implied volatility to incredible levels in order to be in the market, as the Black model would expect diminishing volatility in the price changes at a given implied volatility level for diminishing prices.

A further consequence of dragging the implied volatility curve higher and higher is that the "delta gap," the difference between the Black call delta and the Black put delta, will increase dramatically. An options market maker who sells a put will sell less futures than they should because the Black model will give a higher probability to an up move than a down move. For example, assuming zero interest rates and a futures price of 10, the 0.5 strike has 25% delta for the call, while the put has 75% delta under a 100% vol.[4] If the vol moves to 168%, the delta picture is almost reversed with the calls becoming a 75% delta, while the puts are 25%.

Figure 5
The Explosion of the Front Month WTI Crude (CL) Option Implied Curve Utilizing Black's Model



Note: The implied volatility is quite regular until the next strikes are listed. The curve's put wing (the low strikes) sees a massive move upwards. This move continues until the whole curve reacts to the new reality. Initially, the low Vega of low priced strikes is more important - this fades as the underlying is perceived to be more risky and the limitations of geometric Brownian motion manifest themselves.

Abbreviations: CLH0, March 2020 WTI contract; CLJ0, April 2020 WTI contract; and CLK0, May 2020 WTI contract.

The final issue which hinders the ability of the Black model in these market circumstances is the listing of ***negative*** strike options. Around April 24, 2020, the market started actively trading a zero strike option for the June expiration options. Then -20 strikes were listed, leading to a crescendo which peaked with the trading of the -50 strike option on the June future. The standard Black model cannot be applied to negative strikes without modification. The listing of additional strikes is not unusual. The listing of so many strikes suggests there was demand and a willingness to supply options.

To recap, the industry standard Black model of option pricing has serious difficulties in dealing with zero or negative underlying and strike prices. In addition, the Black model experiences difficulties in pricing options even before reaching the zero boundary. In order to cope, an option seller or a risk manager would have had to sharply increase his/her volatility curves. The increasing of the volatility may result in over- or under-hedging their option risk. We witnessed some of these effects if we looked at the markets through the Black paradigm.

The Alternatives

The problem of negative strikes and negative underlying prices is actually not a new problem in finance and option pricing. This is a problem commonly encountered in power trading and spread option pricing. In the case of power, a point on the electrical grid may pay for power at certain times in the day, at other times it might be paid for consuming power. In recent years, the trading of calendar spread options (CSO) has become quite pervasive. The so-called CSOs are options on the price differential between two expiry months (the calendar spread). In many cases the underlying variable, the spread, can be negative. Moreover, these options are often traded with negative strikes.

Using the spread option model as a guide, we take one very popular model. The model is due Bachelier (Bachelier, 1900), and assumes a simple (additive) Brownian motion, unlike the geometric Brownian motion at the heart of the Black model. This deals immediately with both issues of a zero or negative price and a negative strike.

Bachelier's model is often written as:

$$C_t = \sigma\sqrt{t}e^{-rt}\left(\frac{F-K}{\sigma\sqrt{(t)}}\Phi\left(\frac{F-K}{\sigma\sqrt{(t)}}\right)-\phi\left(\frac{F-K}{\sigma\sqrt{t}}\right)\right) \tag{3}$$

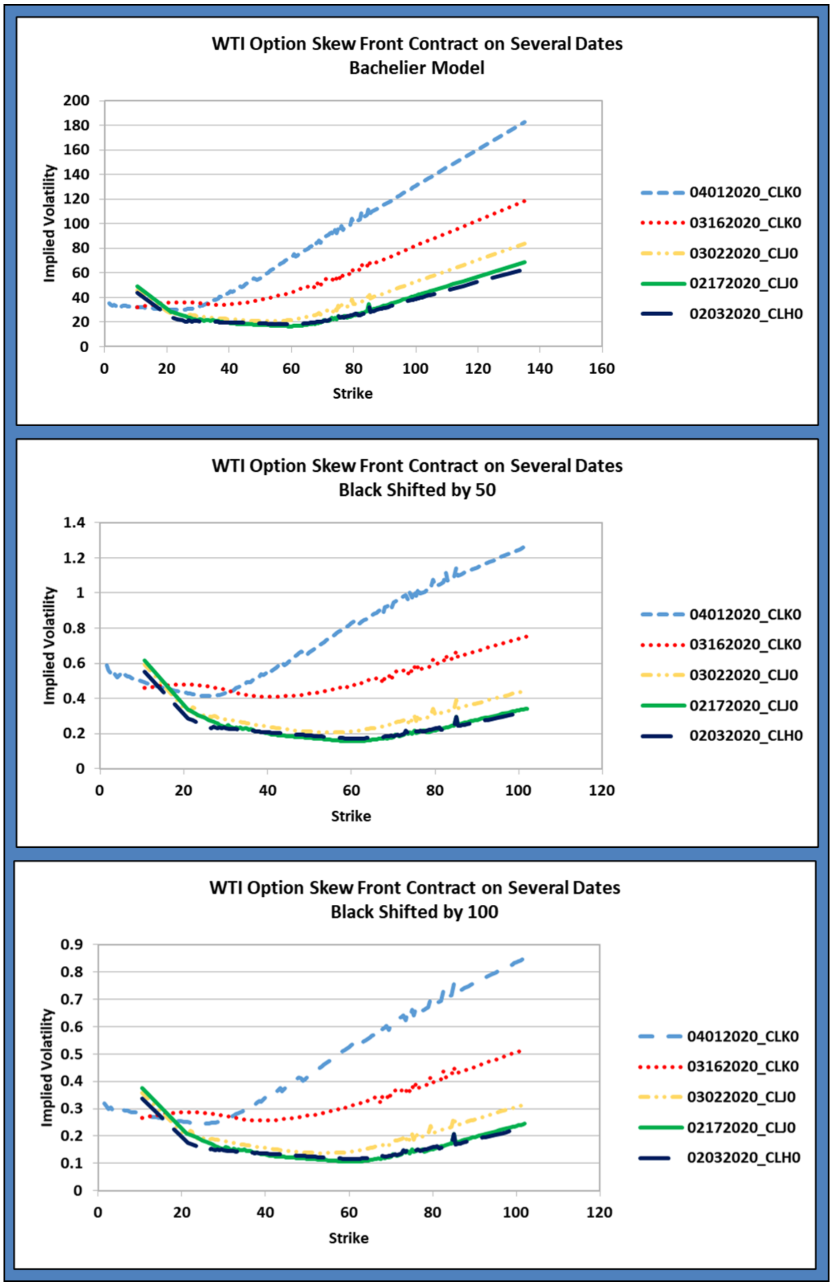
The uppercase phi is the cumulative standard normal while the lowercase phi is the standard normal point density function; all other notations are the same as Eq (1).

There are known modifications that will allow Black's model to price options on products with negative prices. With a very minor change of variables, we can "displace," or shift, the underlying variable of the model. In the case of oil, we could say that the random variable is *M* + the futures price. *M* is an arbitrary constant which is large enough to deal with potential negativity. In order to be consistent, we also shift the strikes by this amount as well.

In Figure 6 on the next page, we show the effect of fitting a standard Bachelier model and Black model with displacements of 50 and 100. The immediate observation that jumps out of the Bachelier model is the difference in the scale of the implied volatility values between the Black and Bachelier. The difference between these two modes is that these models are measuring two different types of price movements. Bachelier measures movement in dollar space and Black measure movements in percentile space. The Bachelier model's implied volatility is on the order of 20-30%. This is in comparison to the Black volatilities, which are 3.0 (or 300%). The implied volatility for the standard Black model appears to be an extreme value, but glancing at Table 1, we see that this is broadly consistent with the annualized standard deviation of differences. What is even more interesting is that when an offset is applied to the Black model, the calculated implied volatility is more consistent with traditional implied volatility values of 0.2 (or 20%) for the longer dated options and 0.60 (or 60%) for the near dated options, which was consistent with the oil market implied volatilities prior to the negative expiring option event.

What is also interesting is that the strong put skew (lower strikes carry a high volatility) is completely eliminated in the displaced Black and Bachelier runs. Furthermore, as we choose a bigger displacement we see a high degree of similarity between the Bachelier view and that of the displaced Black model. This last point is not a totally unexpected outcome. Haug and Taleb (2011) argue that Black's model is a specific application of the Bachelier model's approach towards removing the predictable drift of an underlying variable. By shifting the underlying variable (and strikes) away from 0, we effectively demonstrate their arguments.

Figure 6
Alternative Skews



Conclusion

The move in the May futures price to negative values was historic. The listing of strikes from over +$135 to -$50 was also nothing short of unprecedented (in the June option expiry). The market traded through the 0 dollar boundary with no hesitation. Moreover, the market digested negative strikes with no major breakdowns. This suggests that the market participants were ready for negative prices and negative strikes. Fundamentally, market participants could choose a new model, Bachelier, or stick with a modification of the Black, which is an industry standard. Either competitor (Bachelier and displaced Black) to the industry standard Black model had similar properties. Both exhibited a strong upward slope with respect to the strikes. Both approaches could be used in pricing and hedging. The ease of converting from Black to a displaced Black suggests that most market participants would have made this choice. However, market conditions like this should spur a more generalized view to pricing and risk. In terms of generality, the Bachelier approach has the advantage of being very straightforward.

Endnotes

1 In fact, the other market benchmarks (Brent North Sea, Urals blend and so forth) did not trade negative during this time period. That said, as documented by Blas and Tobben (2020), an "[o]bscure Wyoming crude grade [had been] bid at negative 19 cents a barrel" in mid-March.

2 Some option traders may have priced their June (next month) options using the May future, in which case they would have had to adjust the price using an additive "roll" factor since there was a rather steep upward slope to the price curve at that point.

3 We gratefully acknowledge CommodityVol.com for providing the data for these plots.

4 We assume a 1-year life for simplicity.

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Dr. Greg Sterijevski is the founder of CommodityVol.com, the web's preeminent source of implied volatility and market statistics centered on commodity markets. He is also the co-founder of The Asset Risk Company, the creator of the world's first commercial commodity factor model. Dr. Sterijevski holds a Ph.D. in Economics from the University of Illinois at Chicago. He started his career at the SAS Institute, building statistical techniques and codes for the flagship SAS System. His career took him back to Chicago and its financial markets. There, he helped create and release the first Monte Carlo Margin Model for equity options. He has since held positions in market making options and futures in commodity markets, long-short equity trading among other endeavors. His roles have included: quant, risk manager, partner and trader. The constant has been a model and datacentric approach to risk and trading.

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Dr. Andrew Kumiega has applied his Ph.D. in Industrial Engineering to research positions in both the manufacturing and the financial industry over the last 25 years. He has held multiple Director level positions in financial trading firms responsible for front office financial engineering and risk. He has also held senior positions at financial management firms directing the Quality and IT risk teams.

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